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# Psychological Imprisonment or Intellectual Freedom? A Longitudinal Study of Contrasting School Mathematics Approaches and Their Impact on Adults' Lives

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Abstract: In a previous study of 2 schools in England that taught mathematics very differently, I (JB) found that a project-based mathematics approach resulted in higher achievement, greater understanding, and more appreciation of mathematics than a traditional approach. In this paper Sarah Kate Selling and I describe a follow-up study. Eight years after the students were in school I contacted a sample of the adults from each school to investigate their use of mathematics in life. In this paper we described the ways that the young adults who had experienced the 2 mathematics teaching approaches developed profoundly different relationships with mathematics knowledge that contributed towards the shaping of different identities as learners and users of mathematics (Boaler & Greeno, 2000). The adults from the project-based school, Phoenix Park, had moved into significantly more professional jobs, despite living in one of the lowest income areas of the country, and developed active relationships with mathematics that are impressive. In this article, we consider the different opportunities that the 2 school approaches offered for long-term relationships with mathematics and different forms of mathematical expertise that are differentially useful in the 21st century (Hatano & Oura, 2003).

## Introduction

> *You're putting this psychological prison around them . . . it's kind of . . . people don't know what they can do, or where the boundaries are, unless they're told at that kind of age. (Marcos, Amber Hill)*

The question of the best ways to teach students mathematics continues to be hotly debated around the world (Boaler, 2016; Schoenfeld, 2002; Skemp, 1976; Wilson, 2003), despite the fact that the last 2 decades have produced a wealth of evidence from researchers on productive mathematics learning environments. A number of studies have documented the value of instructional approaches in which students are actively, rather than passively, engaged in mathematics. Active engagement in mathematics, we propose, takes place when students are engaged in problem solving, the discussion of ideas, and application of methods. Passive engagement, we propose, takes place when students are mainly required to listen to a teacher explain methods and solve problems and then reproduce the teacher's methods. Studies have shown the positive effect of active classroom engagement in elementary (Cobb et al., 1991; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), middle (Boaler & Sengupta-Irving, 2016; Schoenfeld, 2002; Silver & Stein, 1996), and high school settings (Boaler, 2015; Boaler & Greeno, 2000; Boaler & Staples, 2008). A related body of research has examined the value of mathematics curricula designed to support active student engagement and the construction of conceptual understanding (see for example, Schoenfeld, 2002; Senk & Thompson, 2003). These studies, which span K–12 settings, have consistently shown that students learning mathematics with active engagement score at higher levels on conceptual assessments than their peers who have experienced more traditional mathematics approaches. Furthermore, these students perform equally as well, or better, on assessments of procedural knowledge. The considerable research body on teaching and learning mathematics provides important evidence pointing to the need to engage students actively in their mathematics learning, both for students' understanding of mathematics and their identities and relationships with mathematics (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009b; Horn, 2008; Langer-Osuna, 2011). Although studies have shown that students' different experiences in schools influence their achievement and identities as learners, few studies have followed students over time (Maher, 2005; Maher, Powell, & Uptegrove, 2011) and examined the long-term effects of different school approaches. We address this issue by reporting on a study of students many years after they completed their secondary school education.

## The Initial Study of Phoenix Park and Amber Hill

In a longitudinal study of two schools in England that taught mathematics very differently, Phoenix Park and Amber Hill, I followed entire cohorts of students (n = 290) through their mathematics classes for 3 years, from age 13 to 16.[1] The different cohorts were matched in terms of social class and prior achievement and had experienced the same school mathematics approaches up to the age of 13. At that point, the students' pathways diverged with the two groups attending schools with very different instructional approaches. Although the students were introduced to the same mathematics content, which was taught by equally qualified teachers using the same national curriculum, the study showed that the students engaged in different practices as they were learning mathematics. Through engaging

[1] Throughout this article, we use I to refer to Jo Boaler, the first author, and we to refer to both authors.

in different learning practices (Ball & Cohen, 1999) the students learned to engage with mathematics differently (Boaler, 2002a, 2002b, 2015). Amber Hill was a comprehensive school that taught mathematics in a fairly traditional way. Beginning at age 13, students were placed into one of eight ability groups for mathematics. They were taught using textbooks, teacher lectures, and practice. Typically, students would sit at desks, teachers would introduce methods for 15–20 minutes, and then students would practice them in their exercise books. Most of the students from Amber Hill described mathematics as boring:

> Steve: The books are a bit boring, the chapters aren't really that good and they repeat the same questions over and over again.
> George: Yes and you only needed to do one to know what's going on.
> (Steve and George, Amber Hill, Year 9)

When students were asked about their mathematics approach, they reported that successful performance depended upon memorization:

> In maths, there's a certain formula to get to, say from a to b, and there's no other way to get to it, or maybe there is, but you've got to remember the formula, you've got to remember it. (Simon, Amber Hill, Year 10)

> In maths you have to remember, in other subjects you can think about it. (Lorna, Amber Hill, Year 10)

Phoenix Park was a very different school. It was also a comprehensive, public school but the teachers held a commitment to project-based teaching and heterogeneous grouping. In mathematics lessons, students worked on open-ended projects that the teachers had designed, and the teachers introduced new content to students only when students needed the knowledge to move forward in their projects (Schwartz & Bransford, 1998). The teachers at Phoenix Park would generally offer a choice between different projects, and students would then choose what they wanted to work on. Some examples of projects that students worked on included: investigating consecutive numbers, exploring different loci in the playground, finding the maximum area of fences, and exploring data and statistics through newspaper articles.

When students were interviewed about this instructional approach to mathematics, they highlighted its openness and the fact that teachers rarely told them what to do but instead asked them probing questions. For example:

> Well I think first of all you have to try it and find your own methods, then if you really get stuck the teacher will come and give you suggestions for stuff and tell you like, how to progress further and then you can kind of think about it. (Andy, Phoenix Park, Year 10)

To understand the students' mathematics experiences, I conducted over 200 hours of classroom observations, administered surveys and interviews each year, and collected data from a range of assessments. My analyses of these data indicated that the project-based approach created different opportunities for mathematics learners. These different opportunities resulted in enhanced mathematical

understanding, higher achievement on tests, and the development of active approaches to knowledge. These were all reflected in students' ability to adapt and apply methods in the various assessments they took (Boaler, 2002a, 2015). After 3 years of working on open-ended projects, the students at Phoenix Park scored at significantly higher achievement levels than the Amber Hill students on a range of assessments, including the UK's national examination (General Certificate of Secondary Education, or GCSE). This was despite the fact the students scored at the same levels on standardized tests 3 years earlier at age 13. The Phoenix Park students also scored at higher levels than the national average, despite being at significantly lower levels when they entered Phoenix Park.

Another important finding from the study was the equitable nature of the Phoenix Park approach—analyses of examination scores showed that there were no achievement differences in mathematics by gender, ethnicity, or social class, an unusual and important achievement for a school. At Amber Hill, typical patterns of social class difference emerged. There was a significant correlation between the social class of students and the ability group into which they were placed - even after controlling for achievement. Investigation of the students who scored at higher or lower levels on the national examination (GCSE) than might be expected from initial achievement showed that most of the high achievers at Amber Hill were middle class and most of the low achievers were working class (Boaler, 1997a, 1997b; 2015).

Part of the reason for the success of the Phoenix Park students related to the active identities (Boaler, 2002c) that they developed as learners and doers of mathematics. Students in the project-based mathematics classrooms of Phoenix Park were asked to use and apply mathematical methods, a process that involved them using their human agency (Pickering, 1995). The students were "required to propose 'theories', provide critiques of each other's ideas, suggest the direction of mathematical problem solving, ask questions and 'author' some of the mathematical methods and directions in the classroom" (Boaler, 2002c, p. 45). This agency proved to be important to their motivation and engagement. When students at Phoenix Park reflected upon their mathematics approach, they highlighted the adaptive ways they used mathematics:

> Well if you find a rule or a method, you try and adapt it to other things, when we found this rule that worked with the circles we started to work out the percentages and then adapted it, so we just took it further and took different steps and tried to adapt it to new situations. (Lindsey, Phoenix Park, Year 10)

During assessments of applied problem solving, the Phoenix Park students consistently out-performed the Amber Hill students through flexible and adaptive thinking (for more detail, see Boaler, 2002a; 2015). One of the main conclusions of the initial study was that the two approaches gave students opportunities to develop different identities as learners, with the majority of the Phoenix Park students developing identities as active mathematics users, and the majority of the Amber Hill students developing identities as passive receivers of knowledge. The aim of the follow up study that we shall now report, was to consider whether these different identities and relationships with knowledge had persisted into their adult lives and changed the ways they used mathematics in their jobs and lives.

## Eight Years Later

Eight years after the students had left their two schools, I contacted the young adults from each school to ask about their employment and invite them to take part in an interview. This follow-up study investigated the question: Did the differing forms of identity and expertise that students had developed at school persist into their working lives and impact their use of mathematics in life? The findings of this study offer a rare opportunity to examine the long-term effect of two contrasting mathematics approaches on students' identities and expertise in mathematics.

## Methods

This follow-up study comprised two parts: a short survey that was sent to all 288 students, now young adults, from the initial study and a set of interviews conducted with 20 of those who replied to the initial survey. The interviews were conducted with only a small number of participants, but the differences in the knowledge use that they revealed made them an interesting and important group to consider, particularly given the scarcity of opportunities to investigate the long-term impact of contrasting and well-documented school mathematics approaches.

### Participants

All students who participated in the initial study were contacted for participation in the current study—181 adults from Amber Hill and 107 from Phoenix Park. The extensive data set collected in the initial study showed that the participants who responded to the survey were representative of the larger school cohorts, both with respect to social class and GCSE achievement. The Phoenix Park students had achieved at significantly higher achievement levels at the end of school, but the 2 samples that responded were equal to each other in terms of achievement. Twenty-two of the participants, 10 from Amber Hill and 12 from Phoenix Park, said that they would be available for interview, and 20 interviews were conducted—10 from each school cohort, with the adults being chosen so that we could achieve gender and achievement balance. The interviews were approximately 1 hour long and were conducted in places that were convenient to the participants, typically, the school they attended or a local coffee shop. All interviews were audio recorded and transcribed. The interviews were semi-structured and they included questions such as:

> Can you describe maths in school?
> Do you like maths now? Did you like maths in school?
> Do you ever use maths in work? How do you use it?
> How useful is the maths you learned in school, in your job now?
> If you are using some maths in your work now, how do you know if you are using it correctly?

More detail on methods, and data analysis are given in Boaler & Selling (2017).

## Results

### Employment and Social Economic Opportunity

Analysis of the employment information provided by the 63 participants revealed significant differences between the two samples. As shown in Table 2, the Phoenix Park participants were working in jobs that were significantly higher on the social class scale than those for the Amber Hill group (t = 2.09, df 63.00, p = 0.04, 96% confidence level)

Table 2
Number (Percentage) of Participants by School in Each Social Class Category, OPCS, 1990

| | Professional 1 | Intermediate 2 | Skilled non-manual 3 | Skilled manual 4 | Partly skilled 5 | Unskilled 6 |
|---|---|---|---|---|---|---|
| Phoenix Park | 0 (0) | 12 (18) | 30 (44) | 15(22) | 11(16) | 0 (0) |
| Amber Hill | 0 (0) | 25 (24) | 36 (35) | 17(16) | 11(11) | 15(14) |

Note. Categories 1, 2 and 3 are typically regarded as middle class, and Categories 4, 5 and 6 as regarded as working class.

A comparison of the social class of the participants to the social class reported by their parents during the initial study revealed that 65% of the Phoenix Park participants had moved upwards in their social class categorization, compared to their parents, whereas approximately half of the Amber Hill participants (51%) were working in jobs with a lower classification than their parents and a further 26% were classified at the same level as their parents had been classified. The Phoenix Park participants exhibited a distinct upward trend in social class that was not evident among the Amber Hill participants (see Figure 1).

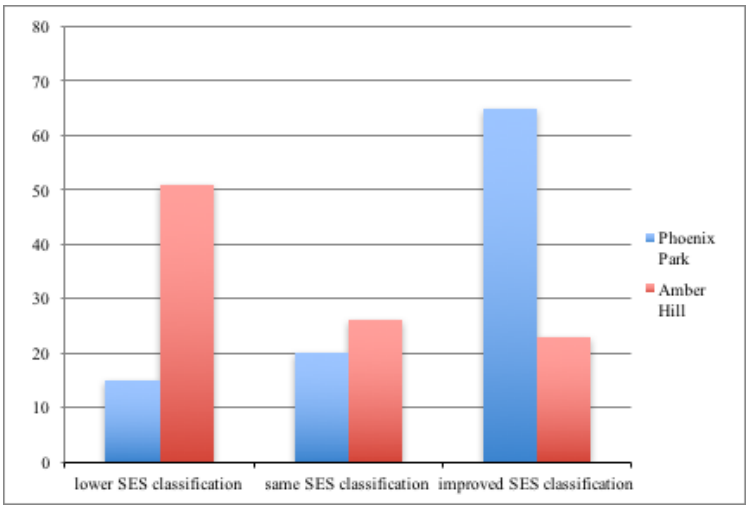



Figure 1. Social class of participants in relation to their parents' placement on OPCS scale at the time of the initial study.

Relationships with Mathematics

The analysis of interview data provided further insight into the employment patterns reported above, suggesting that although the participants scored at comparable GCSE levels, the school experiences of the Phoenix Park participants had given them different relationships with mathematics knowledge, that helped them in work and life. The following findings are organized under the four themes that emerged from the interviews with the two sets of adults.

Satisfaction with school mathematics. At the beginning of the 1-hour interview, the adults were asked: "Can you describe mathematics teaching in your school? What did you do in maths classes?" The adults' descriptions were consistent with the findings that emerged from the initial study of the 2 schools (Boaler, 1998, 2002a). Two of the adults' responses, one from each school, are presented below; these responses typify the descriptions from the two school cohorts.

From Amber Hill:

> You just had to basically turn up for your lesson, have your lesson in front of you, "this is what we're covering today." Like, my language class was a similar thing—parrot-fashion learning, "this is what we're doing today." And basic rules to follow. (Chris, Amber Hill)

From Phoenix Park:

> I always remember . . . you'd work on something for however long it was, and then you would have a discussion with the teacher, and they would perhaps plant another seed in terms of, "Well think about this." And then you'd take that. And it was very much trying to get you to do the thinking. (Simon, Phoenix Park)

In these different statements, the adults highlight the differences between the two teaching approaches, that emerged from the initial study of the two school approaches. At Amber Hill, the students were taught to follow rules: they rehearsed content through short questions, practiced methods they had been shown by their teacher, and used cues from questions to know which method to use. At Phoenix Park, by contrast, the students were given freedom to explore: they learned to ask questions, to inquire using mathematics, and to draw conclusions using mathematical evidence.

The adults were also asked whether they liked maths and whether their school maths classes had been helpful to them in preparing them for their lives. These questions revealed large differences between responses from the two sets of adults. Among the Amber Hill adults, 75% reported enjoying mathematics in their lives, but contrasted their current perceptions of mathematics with those they developed in school. They reported that they saw mathematics all around them in their jobs and lives, and many of them solved mathematical puzzles such as Soduku in their spare time. Sadly, they expressed dismay and confusion that school mathematics had been so uninteresting and unrelated to the mathematics that they now saw around them in life. Sharon represented this view clearly:

> It was never related to real life, I don't feel. I don't feel it was. And I think it would have been a lot better if I could have seen what I could use this stuff for . . . because it helps you, to know why. You learn why that is that, and why it ends up at that. And I think defi-

> nitely relating it to real life is important. (Sharon, Amber Hill)

Chris and Marcos communicated a similar view, highlighting the procedural nature of their mathematics experiences at Amber Hill:

> It's calculating what you need to calculate, and then you do the exam for it, and then it's instantly flushed from your memory because you have no use for it. And it's nothing that's associated with anything to keep it fresh in your mind. It's just a bunch of numbers, you pass the exam, and you're done. (Chris, Amber Hill)

> It was something where you had to just remember in which order you did things, and that's it. It had no significance to me past that point at all. (Marcos, Amber Hill)

Marcos's representation of significance is important because it speaks to the meaning that students may develop, or not develop, when learning school subjects (Damon, 2008). Marcos felt that the mathematics he worked on in school had no or little meaning, which reduced his willingness to engage with the subject.

Whereas the participants from Amber Hill spoke with regret about school mathematics, and three-quarters of them reported enjoying maths now, 100% of the Phoenix Park adults reported enjoyment of maths now and spoke in positive ways when describing their mathematics classes. They reflected upon mathematics lessons, using words such as "brilliant," "ideal" and "brave." As Susan described:

> So I think they had a very good approach to teaching in most subjects. And, as I said, I remember maths being particularly good. I remember the teacher being particularly good also. So I think the way they taught it was fantastic. I remember a lot of people enjoying maths. (Susan, Phoenix Park)

In describing their enjoyment of school mathematics, many of the Phoenix Park participants referred to the openness of the approach:

> I think it was definitely more creative. We were never too much said like "this is going to be on your exam, you need to memorize this." That's another thing that I've had a problem with the education system—just the whole regurgitation just for the exams . I don't know, they might have been prepping us, but it didn't feel like it. (Neil, Phoenix Park)

The stark differences in the adults' reports of their satisfaction with their school mathematics extended to their descriptions of the usefulness of their school mathematics knowledge.

**Using mathematics in life.** When asked about the usefulness of the mathematics they had been taught in school, the adults spoke very differently. All 10 of the adults who had attended Phoenix Park reported that the mathematics they had learned was useful in their jobs, whereas none of the 10 Amber Hill participants thought their school mathematics learning had helped them. The Phoenix Park participants appeared to have moved seamlessly from their mathematics classrooms into the mathematical demands of the workplace, whereas the Amber Hill participants noted a distinct disconnect.

When asked whether school mathematics had been useful, the Amber Hill adults talked about mathematics content. For example, when Trevor, from Amber Hill, was asked if mathematics had been useful to him in life, he said:

> Yeah. I mean stuff like pi and trigonometry, stuff like that. That's never really been useful to me since. I mean I don't think I really remember it anymore. (Trevor, Amber Hill)

Scott spoke similarly:

> I think I've pretty much never used any of it, I think. It's been pretty much me almost teaching myself again the bits that I need to know. I think it needs to be pulled into real-life examples where I would be able to see why I'm calculating what I'm calculating rather than numbers relating to other numbers for no apparent reason. (Scott, Amber Hill)

Helen gave a similar perspective:

> I don't know . . . I mean other things like trigonometry, I don't think you use it really in everyday life, do you? I suppose maybe if you're measuring things and . . . but you don't really use it. (Helen, Amber Hill)

The adults from Amber Hill talked about mathematics in ways that many adults talk about it—as different content areas – numbers, percentages, and trigonometry. The adults from Phoenix Park were distinctive in not doing this; instead, they talked about a subject with many dimensions that went beyond itemized content knowledge. When Andrew was asked whether maths had been useful, he said:

> I suppose there was a lot of things I can relate back to maths in school. You know it's about having a sort of concept, isn't it, of space and numbers and how you can relate that back. And then, okay, if you've got an idea about something and how you would then use maths to work that out. I suppose maths is about problem-solving for me. It's about numbers, it's about problem solving, it's about being logical. (Andrew, Phoenix Park)

What seemed noteworthy about the descriptions from the two groups of participants is not that the participants from Amber Hill said they did not use or had forgotten the content but that they only spoke about content. The Phoenix Park participants talked about mathematics in much broader ways. This reflected the "multidimensional" mathematics (Cohen & Lotan, 2015, Boaler, 2008) that they had learned and the different forms of engagement they had been offered. In the following excerpt, Simon, who had made his way up to senior levels of hotel management, contrasted an approach to education that was focused only on content with his own education at Phoenix Park that had taught him to problem solve and "find ways around" difficulties in work:

> I mean it's like anything. But especially with education, if you look at education on face value, what you're actually taught in school, to me, it just seems complete nonsense. It's the way you manage it, it's the way you apply yourself to it, and the techniques you've learned. You teach yourself, in doing it—that's what I've actually used in the years to

> come. It's not the actual nits and grits of whatever it is—whatever subject—to be honest. Because I think if you struggle at something, you find ways around it, don't you? That's what I took with me. Maybe—maybe—I don't know—but maybe it's the style of teaching then that's given me that. If somebody stood up in a class of 30 and wrote on the blackboard for half an hour and then we did the exercises, maybe I wouldn't think like that, maybe I would just think it's about working out percentages. (Simon, Phoenix Park)

Simon draws a distinction between learning ways of being, which include "teaching" and "applying" yourself, engaging in "struggle" and "finding ways round" difficulties, and learning methods such as "working out percentages."

**Identity and expertise.** The initial study of students in Phoenix Park and Amber Hill showed that Phoenix Park students were more successful in authentic mathematical situations. The students were willing and able to apply their knowledge to solve problems which seemed to relate to their engagement in different practices in school, where they were taught to problem solve and apply knowledge. The Amber Hill students had learned to follow textbook and teacher cues and to repeat procedures, whereas the Phoenix Park students learned to choose from different methods, adapt and apply methods, and draw from resources in their environment (Greeno, 1991). The students had developed opportunities to relate differently to mathematics and to develop different identities as learners (Boaler, 2002b), which seemed to have enabled the development of contrasting forms of expertise in the domain of mathematics (Hatano & Oura, 2003). When the adults in the current study described their jobs, they reflected a very different positioning to knowledge and life, with the Amber Hill adults saying that they did not use school mathematics and they deferred to authorities to know if any mathematical work was correct. The Phoenix Park adults, by contrast, talked about working flexibly with responsibility and agency (Boaler & Greeno, 2000), as we expand upon below. For example Neil, who had worked his way up to a senior level in banking, recalled the responsibility students were encouraged to take in school and the ways this had helped him in his work life:

> I mean I prefer to work in that way now, and that maybe comes from that. Like, you know, I'd much rather work and be given responsibility for doing a job, and not be . . . not have a sort of manager who's always watching what I do and trying to guide me all the time in terms of telling me exactly what to do all the time. I prefer being given responsibility to do something and doing it, and then presenting my results—which is similar to the way we learned in maths. (Neil, Phoenix Park)

One of the interview questions asked the participants to consider a time when they used maths in life or work and to consider how they would know if they were using it correctly. In doing so, Sarah, from Phoenix Park, who was learning to be a teacher, spoke in similar ways about the mathematical responsibility that she had learned in determining the correctness of mathematical solutions:

> Um . . . well I mean I suppose these projects that we did, and the work that we did. If you could prove that the answer that you had, and the solution that you . . . the way that you used to work it out worked for you, and worked with a generic . . you know, if you were able to work it out and prove that it works, that somebody else could also do it, but did it slightly differently. And they were both right. And it wasn't just a cross or a tick, you know?—which is what a lot of maths is. You know, a child faced with a page full of sums,

> and they're either right or wrong in a lot of cases—and that's not always the case in life. (Sarah, Phoenix Park)

Sarah talked about learning that different people may use different methods and solve problems in different ways but also come to the same conclusion. She also stated that what is important in knowing whether solutions are correct is mathematical proof rather than the words of a teacher or book. She contrasted the act of proving an answer with receiving a "cross or tick." This suggests a higher level of mathematical authority and responsibility, something that Neil also referred to when he spoke about presenting ideas in maths class, first as a child and now in his work in banking. Later in the interview, after saying she thought the Phoenix Park approach had prepared her well for life, Sarah was asked what she had learned to do. She described her adaptive approach to mathematics:

> I think maybe looking at . . . being able to look at it from different angles. And also adapting it to other areas. Like, if there's a . . . if I come across a mathematical problem then it's about . . . there's not only one way, or there might not only be one way of sorting it out or working it out. (Sarah, Phoenix Park)

At Phoenix Park, the teachers did not focus only upon mastery of content; their goals were much bigger. They wanted to develop inquiring, problem-solving, and responsible young people. In the interviews with the participants of this study, it seemed that they had achieved this. The former students indicated that they had learned to take intellectual authority as well as a very active stance towards mathematics in their lives (Boaler & Greeno, 2000; Boaler & Sengupta-Irving, 2016; Cobb et al., 2009b; Esmonde, 2009a, 2009b; Langer-Osuna, 2011). In the following excerpt, Sean from Phoenix Park spoke about his approach to examinations, contrasting the approach he learned to one taken by many other students (such as those at Amber Hill) who are taught to follow step-by-step procedures:

> I tend to go into them anyway just thinking that I can do them. Yeah, there wasn't any time I thought, "Well, this is this, this is this." I just worked it out from the knowledge that I had. (Sean, Phoenix Park)

The act of working "it out from the knowledge I had" reflects the development of adaptive expertise and an active approach to mathematics that the students were observed developing in school. Adaptive experts are those who "understand why their procedures work, modify known procedures, or even invent new procedures (Hatano, 1982)" (Hatano & Oura, 2003, p. 28), which seems to be a process Sean was describing.

During the interviews, the adults were asked to describe what they did in a work situation when they encountered some mathematics they could not do. At Amber Hill, the participants talked about getting help from other people, which is certainly a worthwhile strategy. For example, Ian reflected:

> Well if it's at work, then I'm free to approach someone more senior than myself and they'll talk me through it. If it's at home, if I can't work it out for myself, I would try and find out from someone like a friend, or my parents. My mum's fairly clever in maths. (Ian, Amber Hill)

This was not surprising because the Amber Hill students had learned in school that the authority for mathematical correctness lay with teachers and books. At Phoenix Park, the participants again responded very differently when asked the same question:

> JB: What happens if you encounter maths you can't do?
> Clare: Keep on going until I do. Because I wouldn't . . . if something is . . . if I can't do it, then it annoys me. I need to see something through really until the end. So I need to understand how I've done it. If I don't immediately understand it, then I will keep on going at it until I do understand it. (Clare, Phoenix Park)

In these and other interview reflections, the adults from Phoenix Park demonstrated a strong sense of responsibility, agency, and authority (Boaler & Greeno, 2000), and an adaptive form of knowledge they had developed through their active approaches to mathematics (Hatano & Oura, 2003). The adults describe mathematical persistence, which they had been encouraged to develop in their classrooms, with descriptions such as "I will keep on going at it until I do understand it," "I tend to go into them thinking that I can do them," "I prefer being given responsibility and doing it," and "If you struggle at something you find ways around it." These ideas were entirely absent in the descriptions from Amber Hill participants who simply described mathematics as a list of content that had not been useful to them. Further, the Phoenix Park adults stated that they had developed this active, inquiring approach to mathematics in their lives and work from the practices in which they had engaged as school students.

### Intellectual Freedom

When students were learning mathematics at Amber Hill, they were placed into ability groups and taught mathematics that was limited in its scope—students worked on short questions with targeted content. By contrast, the Phoenix Park students worked in heterogeneous groups, and the mathematics was very open. They could take tasks in any direction that they wanted, and the mathematics that they encountered was not predetermined by the mathematics question. There is a common belief (e.g., Oakes & Guiton, 1995) that students will work on appropriate mathematics content if they are in tracked or "setted" groups with students of similar achievement levels. However, in the interviews, it was the adults from Phoenix Park who talked about their work being at the right level for them (as they had also described in school), and the Amber Hill adults talked about the limits placed upon their potential achievement (Hart, Dixon, Drummond, & McIntyre, 2004). In the original study of the two schools many of the students chose to reflect on the impact of ability grouping which became a major finding of the study (see, for example, Boaler, 1997a; 2002a). No question asked about this in the adult interviews, but still seven of the adults chose to talk about constraints or freedom, because of ability grouping. Although the ideas came from only seven of the twenty adults, they seem important to understand. For example, at Phoenix Park, four adults spoke in these ways. One student named Darren said:

> I know in maths, it may have been my own thing that I did quite like maths, but I was always interested in it. And I felt like I could go as fast as I wanted. And you didn't feel like you had to wait for people, or that you were trying to catch up to people. (Darren, Phoenix Park)

Andrew was a high achieving student, who earned the highest examination grade at Phoenix Park. He reflected on the heterogeneity of classes, saying:

> I suppose again everyone was working at different levels. And some people would take some things further than others. But I don't remember having to—you know, if you particularly excel in a subject—having to wait for other people to catch up. (Andrew, Phoenix Park)

Clare and Sarah, both from Phoenix Park, spoke of a different kind of freedom:

> There was a lot more freedom perhaps than in some other schools, than maybe some other schools. It was a lot based . . . umm . . . a lot was left up to the student, and they were expected to think for themselves. (Clare, Phoenix Park)

> JB: What did you most appreciate about Phoenix Park?
> Sarah: Probably the freedom—the freedom to think. (Sarah, Phoenix Park)

The interview excerpts above speak both to the freedom that students experienced to think and to take ideas in different directions as well as the lack of constraints they experienced because they could take work to any level they wanted and work at any speed they wanted.

Three of the Amber Hill adults also spoke in interesting ways, about the constraints placed upon their opportunities for learning the mathematics that interested them:

> I think we could have had a bit more of choice in what we learned in the classroom. Because I feel like I could've learned a lot more stuff that I was interested in, rather than . . . looking back in hindsight, it really hasn't come useful; so I could have spent my time learning more of what I was interested in at the time. (Alan, Amber Hill)

The students also spoke about the limits placed upon their potential:

> We had 1st set, 2nd set, 3rd set—and it kind of . . . well, for me it wasn't necessarily the best experience. So if you were 1st set, you were a maths genius; if you were 2nd set you were very good at maths but it wasn't your best subject. And anything else below that was kind of - you'd choose a job in unskilled labor. (Dave, Amber Hill)

Dave spoke in ways that are reminiscent of Willis' (1977) ethnography of schools in which Willis claimed that students in low ability groups were "learning to labor." Marcos spoke passionately about his perceived disadvantage of students working in setted groups that he had continued to reflect upon in later life:

> You're putting this psychological prison around them . . . it's kind of . . . people don't know what they can do, or where the boundaries are, unless they're told at that kind of age. (Marcos, Amber Hill)

The Phoenix Park adults talked in varied ways about the freedom that they experienced, to work on any mathematics, to think for themselves, and to take work to any level. Their words speak of an intellectual freedom that other students have described when experiencing open mathematics with growth mindset messages (see Boaler, 2016). Students in California, from different grade levels, who had moved to a more open mathematics approach described feeling "free," "open" and "alive" (Boaler, 2016, p. 189). The Phoenix Park adults seemed to share a similar sense of intellectual freedom, which was ironically matched in its strength of feeling by the Amber Hill adults' perceptions of constraints. Marcos captured his sense of constraint vividly, describing the messages that ability grouping sends as a form of "psychological imprisonment."

## Discussion and Conclusion

The combined forms of data collected in the study strongly suggest that the Phoenix Park adults, with their very different mathematical experiences in school, were given a head start in life. Although it is difficult to separate the influence of the students' whole school experiences from their mathematical experiences, the students' whole school experiences varied along similar dimensions. Amber Hill was a traditional school in which most subjects were taught traditionally. Amber Hill employed ability grouping across the school, although mathematics divided students into the most groups (eight). Phoenix Park was a progressive school that was proud of its tradition of giving students' responsibility and employing project-based teaching methods across the school.

One explanation for the advancements in employment and social class from students who attended Phoenix Park might be the affluence of the two areas and the job opportunities provided in the different locales in which the young adults lived, but this hypothesis cannot be supported by the differences in areas in which they lived. Both sets of participants had remained in their local area, in both cases, low-income areas of England. However, the Amber Hill participants lived in an area of higher social advantage because it bordered a large and affluent city with a much wider range of jobs available to them. In contrast, Phoenix Park is situated in a lower resourced area, and most of the participants who attended the school lived on the same housing estate (similar to what is called a "project" in the United States) when they returned surveys. The explanation we offer: that their different school experiences gave the Phoenix Park students a better start in life and afforded them the opportunity to move upward in the social scale—seems likely. Indeed, this small but representative data set would suggest that Phoenix Park, a progressive school in one of the lowest income areas of the country, helped the students to become upwardly mobile. The interview data supported and gave further insights into this phenomenon.

The interviews from this study represent a small sample, but the extreme differences communicated by the two sets of young adults as they talked about mathematics make them important to consider. At Phoenix Park, the students had learned to engage in practices that were very different from those at Amber Hill. These involved the students acting with agency and authority—asking questions, choosing mathematical directions, and determining the correctness of their work. By contrast, the Amber Hill students, who had learned mathematics content and performed very well in classroom exercises, engaged with mathematics passively, learning to practice content by rehearsing methods and checking for correctness with the teacher or the book. These ways of engaging seemed to have had an important effect on the young people as adults, as they described interacting with knowledge and work tasks differently. In interviews, the Amber Hill participants communicated frustration with their school maths approach and dismissed most of school mathematics as irrelevant to their work and lives. By contrast,

the Phoenix Park adults talked with confidence about tackling any problem they encountered and seeing mathematics as knowledge that they could adapt and use. They were prepared to "keep on going at it," "struggle," "find ways round" problems, and take "responsibility." Their words seem to reflect actions and beliefs that combine in the development of more active and capable mathematical identities, growth mindsets (see Boaler, 2016), and adaptive expertise. These differences seem likely to explain, at least in part, the Phoenix Park participants' greater advancement in life with jobs that were significantly higher on the social class scale.

The adults who were interviewed in this study had achieved at the same levels on national examinations, but it was clear that they had learned to relate to mathematics knowledge very differently. This finding highlights the limitations of achievement tests as indicators of students' competency and knowledge use. Hatano and Oura (2003) have outlined 2 forms of expertise. They describe "routine expertise" as being able to "solve familiar types of problems quickly and accurately", but not going beyond procedural efficiency. They describe '"adaptive expertise" as a range of "flexible, innovative, and creative competencies" (p. 28) which allow students to propose and modify procedures, apply their ideas and respond flexibly to different situations. Hatana and Oura speak to the need for today's students to develop adaptive expertise, which seems to match the capabilities developed by the students at Phoenix Park .

Learning environments in which students interact with mathematics actively and engage in a broad range of mathematical practices are still relatively rare in mathematics classrooms (Jacobs et al., 2006; Litke, 2015). The reflections of the Phoenix Park and Amber Hill participants suggest that such experiences may not only enhance individual understanding but also provide students with opportunities to develop adaptive expertise and to engage successfully with mathematics in their lives. At Amber Hill, the students had learned formal mathematical methods, but they had learned them through passive participation structures, and in later years, they dismissed the methods as irrelevant. In talking with the Amber Hill adults, it seemed that their mathematical identities included submission to outside authorities—canons of knowledge and lists of content. By contrast, Phoenix Park adults talked about actively using knowledge to solve problems. They saw mathematics as knowledge that they could use, adapt, and apply to different situations. The equity focused classrooms of Phoenix Park, which included encouraging all students towards high achievement, appeared to be reflected in the positive reports of the Phoenix Park adults and their success in life. It seems very likely that the differences communicated by the adults contributed toward the Phoenix Park adults moving up the social scale and gaining more success than the Amber Hill adults in their work and life.
Martin and Schwartz (2009) point to the importance of adaptive expertise in 21st century work and remind us of the value of being able to adapt in changing circumstances. They also caution that there have been too few studies of the development of adaptive expertise or understanding of "the characteristics of adaptive behaviors or the conditions that lead to them" (Martin & Schwartz, 2009, p. 370). The study of Phoenix Park students in school and this longitudinal study of some of the adults later in their lives may be a useful resource in helping us understand the ways teachers may help students develop adaptive expertise as well as positive and active mathematical identities that students will need as they enter 21st century employment (see Boaler, 2013, 2016).

Various research studies show the importance of active mathematics engagement in classrooms and the ways that schools and teachers may provide such learning environments (e.g., Boaler & Greeno, 2000; Cobb et al., 1991; Silver & Stein, 1996). As we move into an increasingly technological world that

requires both mathematical competence and adaptive expertise, the need to shift mathematics classrooms in these directions becomes ever more urgent (Wolfram, 2010). If mathematics classrooms do not engage students actively in mathematics learning, giving positive messages and opportunities to all students (Boaler, 2016), they may forever be reminding students of the mismatch between the mathematics that they learn in school and the mathematics that they need for today's innovative, adaptive, and technological world. Providing opportunities for all students to develop active mathematical identities and adaptive expertise is extremely important for the future of our society (Boaler, 2016). It is our hope that research studies of teachers and students working in these ways will enable many more mathematics teachers to pursue this worthy goal.

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